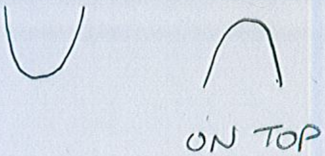
Find the center of mass of the region between the curves $y = 11x^2$ and $y = 12x - x^2$. **SCORE: _____ / 14 PTS**

$11x^2 = 12x - x^2$

$12x^2 - 12x = 0$

$12x(x-1) = 0$

$x = 0, 1$ (1)




$\int_0^1 (12x - x^2 - 11x^2)\,dx = \int_0^1 (12x - 12x^2)\,dx = (6x^2 - 4x^3)\Big|_0^1 = 2$ (1) (1½) (1)

$\int_0^1 x(12x - 12x^2)\,dx = \int_0^1 (12x^2 - 12x^3)\,dx = (4x^3 - 3x^4)\Big|_0^1 = 1$ (1) (1½) (1)

$\frac{1}{2}\int_0^1 ((12x - x^2)^2 - (11x^2)^2)\,dx = \frac{1}{2}\int_0^1 (144x^2 - 24x^3 - 120x^4)\,dx$ (1½)

$= \frac{1}{2}(48x^3 - 6x^4 - 24x^5)\Big|_0^1 = 9$ (1½) (1)

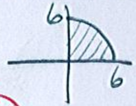
$\frac{1}{2}(1, 9) = \left(\frac{1}{2}, \frac{9}{2}\right)$ (2)

The time for a tax refund check to be deposited after it has been received is a continuous random variable $X$ **SCORE: _____ / 10 PTS**

(in units of days) with probability density function $f(x) = \begin{cases} k\sqrt{36-x^2}, & x \in [0,6] \\ 0, & x \notin [0,6] \end{cases}$ for some constant $k$.

Find the mean (average) time before a tax refund check is deposited.

$\int_0^6 k\sqrt{36-x^2}\,dx = 1$ $1\frac{1}{2}$




$k\left(\frac{1}{4}\cdot\pi(6)^2\right) = 1$

$k = \frac{1}{9\pi}$ $1\frac{1}{2}$

$1\frac{1}{2}$ $\int_0^6 \frac{1}{9\pi}x\sqrt{36-x^2}\,dx$

$u = 36-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$x=6 \rightarrow u=0$

$x=0 \rightarrow u=36$

$3$ $= \int_{36}^{0} \frac{1}{9\pi}\left(-\frac{1}{2}\sqrt{u}\right)du$

$1$ $= -\frac{1}{\cancel{18}_{9}\pi}\frac{\cancel{2}}{3}u^{\frac{3}{2}}\Big|_{36}^{0}$

$= \frac{1}{27\pi}\cdot 36^{\frac{3}{2}}$

$= \frac{1}{27\pi}6^3$

$= \frac{8}{\pi}$ DAYS

$1\frac{1}{2}$

Find the length of the parametric curve $\begin{array}{l} x = 1 + 4t^{\frac{3}{2}} \\ y = \frac{3}{4}t^3 - 4\ln t \end{array}$ for $1 \le t \le e$. **SCORE: _____ / 6 PTS**

$$\int_1^e \underbrace{\sqrt{\left(6t^{\frac{1}{2}}\right)^2 + \left(\frac{9}{4}t^2 - \frac{4}{t}\right)^2}}_{2}\,dt$$

$$= \int_1^e \sqrt{\left(36t + \frac{81}{16}t^4 - 18t + \frac{16}{t^2}\right)}\,dt$$

$$= \int_1^e \sqrt{\underbrace{\frac{81}{16}t^4 + 18t + \frac{16}{t^2}}_{1}}\,dt$$

$$= \int_1^e \underbrace{\left(\frac{9}{4}t^2 + \frac{4}{t}\right)}_{1} dt$$

$$= \underbrace{\left(\frac{3}{4}t^3 + 4\ln t\right)}_{1}\Bigg|_1^e = \frac{3}{4}e^3 + 4 - \frac{3}{4} = \underbrace{\frac{3}{4}e^3 + \frac{13}{4}}_{1}$$